



## OPEN ACCESS

### EDITED BY

Abhishek Kumar Srivastava,  
Indian Institute of Technology  
(BHU), India

### REVIEWED BY

João M. da Silva Santos,  
National Solar Observatory,  
United States  
Jenny Rodriguez Gomez,  
National Aeronautics and Space  
Administration, United States

### \*CORRESPONDENCE

David B. Jess,  
✉ [d.jess@qub.ac.uk](mailto:d.jess@qub.ac.uk)

RECEIVED 06 May 2026

REVISED 13 August 2026

ACCEPTED 18 August 2026

PUBLISHED 23 September 2026

### CITATION

Duckenfield TJ, Jess DB, Jafarzadeh S,  
Roupe van der Voort L and  
Henriques VMJ (2026)  
Formation-height diagnostics across a  
sunspot: a comparison of Lightweaver  
radiative transfer and NICOLE inversions  
using the Ca ii 8542Å line.  
*Front. Astron. Space Sci.* 13:1873920.  
doi: 10.3389/fspas.2026.1873920

### COPYRIGHT

© 2026 Duckenfield, Jess, Jafarzadeh,  
Roupe van der Voort and Henriques.  
This is an open-access article  
distributed under the terms of the  
[Creative Commons Attribution License](https://creativecommons.org/licenses/by/4.0/)  
(CC BY). The use, distribution or  
reproduction in other forums is  
permitted, provided the original  
author(s) and the copyright owner(s) are  
credited and that the original  
publication in this journal is cited, in  
accordance with accepted academic  
practice. No use, distribution or  
reproduction is permitted which does  
not comply with these terms.

# Formation-height diagnostics across a sunspot: a comparison of Lightweaver radiative transfer and NICOLE inversions using the Ca ii 8542Å line

Timothy J. Duckenfield <sup>1</sup>, David B. Jess <sup>1,2\*</sup>,  
Shahin Jafarzadeh <sup>1,3</sup>, Luc Roupe van der Voort <sup>3,4</sup> and  
Vasco M. J. Henriques <sup>3,4</sup>

<sup>1</sup>Astrophysics Research Centre, School of Mathematics and Physics, Queen's University Belfast, Belfast, United Kingdom, <sup>2</sup>Department of Physics and Astronomy, California State University Northridge, Northridge, CA, United States, <sup>3</sup>Rosseland Centre for Solar Physics, University of Oslo, Oslo, Norway, <sup>4</sup>Institute of Theoretical Astrophysics, University of Oslo, Oslo, Norway

The Ca ii 8542Å line is widely used by modern instruments such as SST/CRISP and DKIST to probe the highly structured and dynamic chromosphere. However, interpreting observations in this line requires a characterisation of the effective height ranges sampled by each spectral tuning, particularly when geometric height is obtained from inversion atmospheres that assume hydrostatic equilibrium. Using high resolution observations of a sunspot, we present a direct comparison between formation-height diagnostics derived from non-LTE radiative-transfer calculations and the geometric heights returned by NICOLE inversions, covering atmospheres corresponding to the umbra, umbral-penumbral boundary, penumbra, and the penumbral-quiet sun boundary. Formation-height probability distributions are computed from Lightweaver contribution functions evaluated in the inversion atmospheres, and convolved with the SST/CRISP transmission profile. We then identify the optical-depth layers whose associated geometric-height distributions from the inversions best match these formation-height probability distributions. We find that the optical-depth range sampled by the line remains broadly stable across the sunspot, whilst the corresponding geometric heights vary systematically in the radial direction. Across the modelled annuli, the selected optical depth intervals remain broadly stable, whilst the corresponding model geometric heights vary with radius. Within the adopted inversion framework, the resulting mapping relates CRISP spectral tunings to optical depth and model geometric height and provides a practical guide for interpreting Ca ii 8542Å observations of chromospheric waves and dynamics, particularly in the era of next-generation facilities such as DKIST.

### KEYWORDS

chromosphere, radiative transfer, spectropolarimetric inversions, Sun: magnetohydrodynamics, sunspots

# 1 Introduction

Probing and modelling the solar chromosphere is essential to understanding one of the major unresolved questions in solar physics: how energy is transported and coupled between the photosphere and the atmospheric layers above it. Magnetohydrodynamic waves serve as remote probes into the physics of these otherwise inaccessible areas, as well as being likely candidates themselves for mediating the energy transport between layers (Jess et al., 2015; Jess et al., 2023). However, interpreting such waves in the dynamic and complex chromosphere is made difficult by the way such spectral diagnostics sample atmospheric height (Murabito et al., 2019). It is common for chromospheric lines to not be in local thermal equilibrium (non-LTE), meaning the emerging intensity is not determined primarily by the underlying plasma temperature, which would be the case for most photospheric lines (Carlsson and Stein, 1997). Instead radiative scattering can have an important effect, meaning the intensity at a given wavelength is produced from a broad vertical extent of the atmosphere, not any one specific height (Leenaarts et al., 2012). Additionally, different spectral positions will reflect different regions of contribution, and further complexity is introduced by the velocities known to be prevalent in the chromosphere, leading to substantial Doppler shifts and hence change the effective formation height, through opacity shifts (Noda et al., 2016; Cauzzi et al., 2008). Since the emergent intensity at a given wavelength is typically most sensitive to layers where the optical depth is of order unity, spectral diagnostics are naturally interpreted in terms of their optical-depth (often parameterised using the reference scale  $\log \tau_{500}$ , defined with respect to the continuum opacity at 500 nm). However, the strong stratification of the atmosphere changes the mapping between optical depth and geometric height across the field of view, essentially meaning the same spectral position in a line can probe different depths at different locations.

The Ca ii 8542Å infrared triplet is one of the principal chromospheric probes used in high resolution observations made commonplace today from facilities such as SST/CRISP (Scharmer, 2006; Scharmer et al., 2008) and DKIST (Rimmele et al., 2020), due to its strong sensitivity to chromospheric thermodynamics and magnetic structure, combined with the fact that in typical chromospheric conditions, calcium is mostly singly ionised, where non-equilibrium and partial redistribution effects can be neglected (Cauzzi et al., 2008; Noda et al., 2016; Díaz Baso et al., 2019). Nevertheless, relating the observed signals to their originating geometric formation heights is non-trivial. Spectropolarimetric inversion techniques such as those implemented in the NICOLE code (Socas-Navarro et al., 2015) provide stratified atmospheric models as a function of optical depth. These inversion atmospheres are obtained by iteratively solving the radiative-transfer equation to best reproduce the observed spectra. Subsequent forward radiative-transfer calculations can then be used to determine how the line samples those atmospheres as a function of wavelength and height. Given the advent of next-generation solar facilities such as DKIST enabling routine high-resolution measurements of chromospheric structure and dynamics (Rimmele et al., 2020), it is increasingly important to determine, within clearly stated model assumptions, how reliable inversion-derived atmospheric stratifications are when compared

to the geometric height ranges sampled by spectral diagnostics used in spectroscopic analyses of wave signatures, an issue that has not yet been examined in detail. Furthermore, the challenge of relating inversion-derived optical depth to geometric height is not unique to the chromosphere, but also arises in photospheric studies (e.g., Rodríguez-Gómez et al., 2024).

In this work, using stunning observations of a non-flaring sunspot in the Ca ii 8542Å line, we compare the atmospheric stratifications inferred from NICOLE inversions (Socas-Navarro et al., 2015) with the formation-height distributions derived from radiative-transfer calculations performed with Lightweaver (Osborne and Milić, 2021). By performing this comparison across a sequence of radial annuli spanning a sunspot atmosphere, we assess how Ca ii 8542Å samples the chromosphere and how inversion height scales relate to the height sensitivity of the spectral diagnostic.

# 2 Observations

The observations used in this study were taken by the CRISP Imaging Spectropolarimeter (CRISP; Scharmer et al., 2008) at the Swedish 1-m Solar Telescope (SST; Scharmer et al., 2003) at the Observatorio del Roque de los Muchachos, La Palma, Spain. The dataset was acquired on 29 April 2016 between 09:42 and 12:08 UT, targeting a sunspot in NOAA AR 12533 at helioprojective coordinates  $(x, y) = (623'', 19'')$  corresponding to the cosine of the heliocentric angle  $\mu \approx 0.75$ . The CRISP data have a pixel scale of  $0.059'' \text{ pix}^{-1}$  and cover an original field of view of approximately  $60'' \times 60''$ . The time series comprises 431 consecutive CRISP scans with a temporal cadence of 20.3 s. The CRISP spectral tunings sample the Ca ii 8542Å line at 21 wavelength positions spanning the inner wings and line core, with offsets extending to  $\pm 1750 \text{ mÅ}$  from the line centre, sensitive to atmospheric layers from the upper photosphere to the chromosphere. The data were reduced using the CRISPRED pipeline (de la Cruz Rodríguez et al., 2015), including image restoration with the multi-object multi-frame blind deconvolution (MOMFBD; van Noort et al., 2005) technique. These observations are part of a coordinated SST-IRIS campaign; a broader overview of such coordinated datasets is provided by Rouppe van der Voort et al. (2020).

The Doppler-averaged data were obtained by averaging the intensity images at the two outer spectral sampling positions ( $\pm 0.945 \text{ Å}$  from the line core) at each time step, thereby reducing sensitivity to Doppler shifts. The resulting data were normalised by the quiet-Sun intensity to place the observations on a common relative intensity scale, allowing consistent intensity thresholds to be used when defining the umbral and penumbral boundaries. The penumbral and umbral boundaries were determined by fitting ellipses to the normalised intensity contours of 0.95 and 0.50, respectively, as illustrated in Figures 1, 2. To investigate how formation-height properties vary across the sunspot atmosphere, the field of view was divided into a sequence of concentric elliptical annuli centred on the sunspot umbra. These annuli follow the observed intensity shape of the sunspot and divide it into regions corresponding (approximately) to: the inner umbra; outer umbra; the umbral-penumbral boundary; inner penumbra; outer penumbra; and penumbra-quiet-Sun boundary.

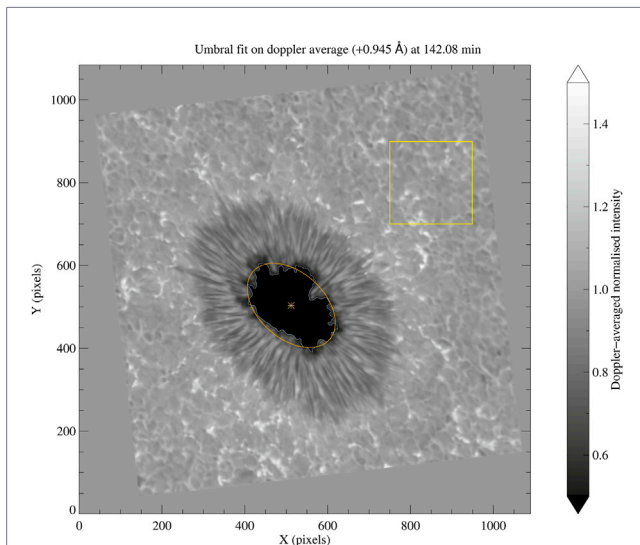


**FIGURE 1**  
The sunspot observed by SST/CRISP over 2 h 26 min. The image is formed by averaging the blue- and red-wing intensities at  $\pm 0.945 \text{ \AA}$  from the line core, and has been normalised by the quiet Sun intensity averaged across the yellow box. The orange ellipse has been fitted to the normalised intensity contour value of 0.5 corresponding to the umbral boundary. This figure is available as a video. The six annular regions used in the subsequent analysis are shown and individually labelled in Figure 2.

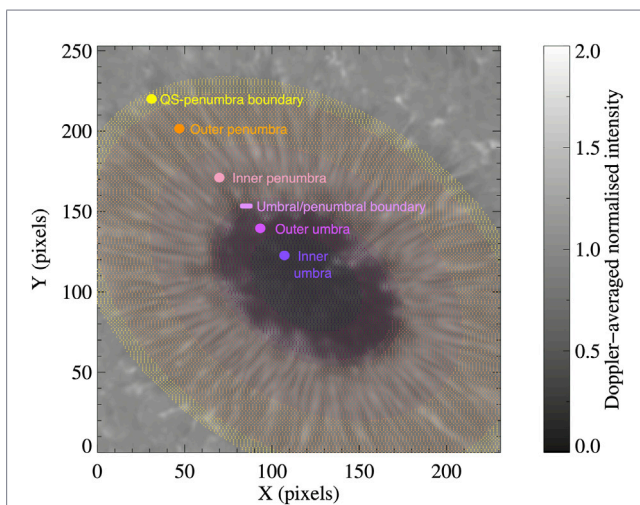


**FIGURE 2**  
Doppler-averaged and quiet-Sun normalised intensity map of a sunspot showing the six concentric elliptic annuli used for spatial averaging and atmospheric stratification analysis, ordered from the inner umbra to the penumbra-quiet-Sun boundary. The field of view corresponds to the pixels for which inversions were calculated.

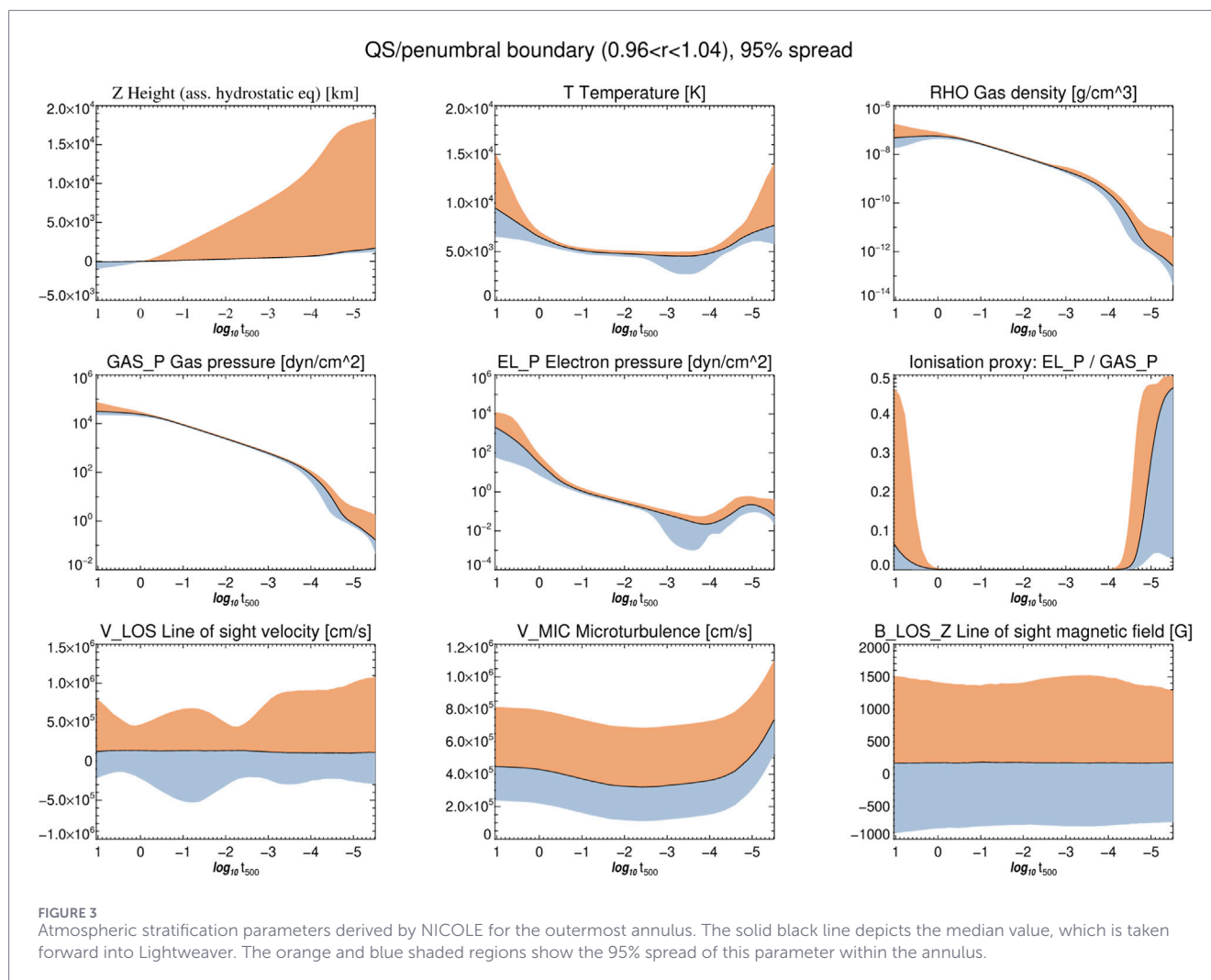
For a slightly reduced field of view centred on the umbra (macro-binned grid of  $231 \times 253$  pixels<sup>2</sup>), the full-Stokes Ca ii 8542Å profiles were inverted with the non-LTE inversion code NICOLE (Socas-Navarro et al., 2015) to retrieve the depth-dependent atmospheric stratification as a function of optical depth. We employed a five-level (plus continuum) Ca ii atom and used a semi-empirical atmosphere as the initial model. The

inversions were carried out in multiple cycles with progressively increasing node numbers and intermediate smoothing, iteratively minimising the difference between the observed and synthesised Stokes profiles. Figure 2 shows the spatial locations of the annuli overlaid on the reduced macro-binned grid for which inversions are available. The same annular masks are used throughout the analysis to extract representative inversion atmospheres and to compute formation-height statistics. We note that the CRISP instrumental transmission profile was included when synthesising the profiles used by NICOLE, ensuring that the observed and modelled spectra were compared at the instrumental spectral resolution.

For each region, the percentile profiles of the inversion-derived atmospheric parameters are extracted as a function of optical depth to characterise the range of atmospheric conditions represented within the annulus. These include the temperature, density, gas pressure, electron pressure, electron-to-gas pressure ratio (used here as an ionisation fraction proxy), line-of-sight velocity, microturbulent velocity, line-of-sight magnetic field, and geometric height. The inversion-derived atmospheric conditions (median and 95% spread) for the penumbral-QS boundary is shown in Figure 3 as an example—the other regions may be found in Supplementary Appendix 1.

It is worth noting the distribution of geometric heights returned by the NICOLE inversions become increasingly broad towards lower optical depths. Moreover, while the median height remains within expected chromospheric values (below 1800 km for all annuli over the range considered), the upper envelope of the 95% interval reaches heights an order of magnitude greater than typical reported formation heights of Ca ii 8542Å of some 1,500 km (Cauzzi et al., 2008), e.g., see top left panel of Figure 3. Here we regard inversion heights of 10,000 km or more as implausible for Ca ii 8542Å in this data set, since they exceed commonly reported line core formation heights by more than an order of magnitude. In all regions there exists a secondary population of pixels with implausibly large (inversion) heights at low optical depth (in this dataset, for  $\log \tau_{500} \leq -5.6$ ), which increases as  $\log \tau_{500}$  decreases, despite no evidence of density enhancements associated with chromospheric material at these heights. This spurious population likely stems from the limited sensitivity of the Ca ii 8542Å line to local thermodynamic variations and structure at low optical depths (beyond  $\log \tau_{500} \sim -4.5$ ), where the inferred atmospheric stratification becomes increasingly weakly constrained. This in turn results in an increasingly unreliable  $\tau$ - $z$  mapping under the assumption of hydrostatic equilibrium. This does not preclude the existence of chromospheric material or structures at such heights; rather, we find no indication from these observations nor the corresponding inversion-derived atmospheres that the Ca ii 8542Å line receives a significant contribution from those layers.

Residual observational effects, including finite spatial resolution, the instrumental point-spread function, and atmospheric seeing, may contribute to the overall inversion uncertainty, but their influence has not been quantified here. The anomalous heights occur where Ca ii 8542Å provides weaker constraints on the atmospheric stratification and are therefore consistent with uncertainties in the inversion-derived hydrostatic  $\tau_{500nm}$ - $z$  mapping. The relative contributions of observational and inversion-related effects cannot be determined from the present analysis.



### 3 Methods

To compare the height sensitivities of the Ca ii line for each annulus, radiative-transfer calculations are performed using the modern *Lightweaver* code. *Lightweaver* solves the non-local thermodynamic equilibrium radiative-transfer problem for a specified one-dimensional atmosphere and returns both emergent intensities and wavelength-dependent contribution functions (Osborne and Milić, 2021). The contribution function describes the relative contribution of different heights in the atmosphere to the emergent intensity at a given wavelength, providing the basis for relating the CRISP spectral tuning positions to atmospheric height. *Lightweaver* shares its underlying non-LTE solution strategy and atomic modelling heritage with RH (Uitenbroek, 2001) and has been benchmarked against that code (Osborne and Milić, 2021). The present results nevertheless depend on the adopted atomic models and numerical setup, as discussed below.

For each annulus, the median inversion atmosphere is used as input to the radiative-transfer calculations, to provide a representative stratification for that region while reducing the influence of outliers and unphysical structure in individual pixel inversions. Synthetic Ca ii 8542Å spectra are thus computed

using the median atmospheric structure, over a dense wavelength grid spanning the observed spectral sampling of the CRISP instrument. The radiative-transfer solution was obtained in non-local thermodynamic equilibrium (nLTE) including hydrogen and calcium as active atomic species, with the electron density taken self-consistently from the inversion atmosphere and charge conservation enforced throughout the statistical-equilibrium calculation. Including hydrogen in the statistical-equilibrium calculation is important because it provides a dominant contribution to the electron density in the chromosphere, which in turn strongly influences the ionisation balance and level populations of the Ca ii atoms. Solving for both species simultaneously therefore makes a more physically consistent estimate of the calcium line response, particularly in the middle and upper chromosphere where departures from LTE are significant. The NICOLE inversion atmospheres were not obtained using the same atomic treatment: hydrogen and calcium were treated as active species in the *Lightweaver* calculation, but not equivalently in the inversions. Consequently, the *Lightweaver* populations and contribution functions are not strictly self-consistent with the inversion calculation that produced the input atmospheres. The magnitude of the resulting difference has not been quantified here

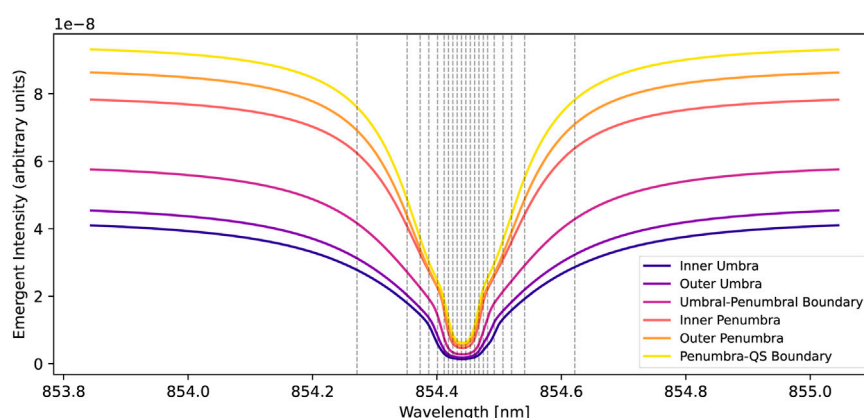


**FIGURE 4**  
Emergent intensity calculated for the Ca ii 854.2 nm line using Lightweaver for six radial annuli. The vertical dashed lines indicate the SST/CRISP spectral positions.

and therefore represents an additional source of model dependence in the inferred formation-height distributions.

## 4 Results

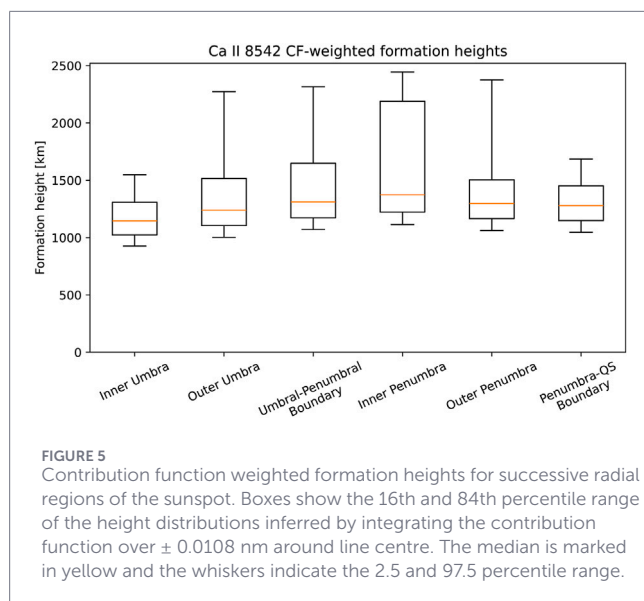
Figure 4 shows the emergent (vertical) intensity profiles obtained from the radiative transfer synthesis for the different annuli. The profiles illustrate both the systematic increase in line-core intensity and the changing wing behaviour with radial distance from the umbral centre, in line with expectations of the changing thermodynamic structure of the atmosphere from umbra to penumbra and previous observations of the Ca ii line profile (Cauzzi et al., 2008; Díaz Baso et al., 2019).

For each wavelength position, the contribution function describes the relative contribution of different atmospheric layers to the emergent intensity. The wavelength-dependent contribution functions returned by the radiative-transfer calculations are then convolved with the SST/CRISP instrumental transmission profile at each spectral tuning position (i.e., the vertical lines in Figure 4). Because chromospheric spectral lines form over extended height ranges, it is convenient to interpret the contribution function as a probability distribution in geometric height, representing the relative contribution of each atmospheric layer to the emergent intensity. Thus these instrument-weighted contribution functions were then normalised in geometric height at each wavelength to produce formation-height probability density functions (PDFs), describing the range of atmospheric layers sampled by the spectral diagnostic.

To illustrate the variation in formation-height sensitivity between annuli, representative statistics derived from the instrument-weighted contribution functions are summarised in Figure 5 using a box-and-whisker representation. Several interesting features are immediately apparent. The median formation height of the Ca ii 8542Å line core varies only weakly across the sunspot, increasing moderately from approximately 1,100 km in the inner umbra to 1,400 km in the inner penumbra, before decreasing slightly toward the outer edge. The lower

percentile ranges remain nearly parallel across all annuli, indicating that the lower boundary of the line's chromospheric sensitivity is largely invariant with radial position. In contrast, the upper percentile ranges show substantially greater variation, particularly across the umbra-penumbra transition and within the inner penumbra, where the vertical extent of the formation-height distribution increases markedly. This behaviour indicates that the main radial differences in the effective formation height comes from changes in the high-altitude tail of the contribution function, rather than from a substantial shift in the bulk of the distribution. That is to say, the median formation height changes relatively little across the sunspot, whereas the upper tail of the formation-height distribution shows a stronger radial variation, as reflected by the 84th and 97.5th percentile formation heights. The outermost boundary region exhibits a narrower distribution again, resembling the umbral case but displaced slightly upward overall, suggesting that the enhanced vertical spread of sensitivity is localised to the intermediate penumbral atmosphere, rather than increasing monotonically with radius.

One can contrast the formation heights for Ca ii 8542Å with the formation heights for H $\alpha$  reported in Leenaarts et al. (2012), which in their case finds a narrow contribution at 400 km and a much broader multi-peaked contribution higher up at 1.0–2.5 Mm. This difference reflects the more strongly scattering-dominated non-LTE formation of H $\alpha$ , whereas the Ca ii 8542Å infrared line remains primarily sensitive to more confined mid-chromospheric layers, making the two lines complementary diagnostics of the chromosphere. One could similarly compare these results with those for the Mgii h and k lines reported by Leenaarts et al. (2013), whose line centre(s) are found to have the optical-depth of unity near  $\approx 2$  Mm. However, their strongly scattering-dominated non-LTE formation means that a single geometric formation height is not meaningfully defined, instead reflecting an extended upper-chromospheric response and a much weaker mapping between optical depth and height than found here for Ca ii 8542Å (discussed further below). Looking lower down, Wu et al. (2025) find formation heights of roughly 0.1–0.5 Mm in the upper photosphere for the Mgi 12.32  $\mu$ m line; its formation is much closer to LTE, and therefore



has a more direct mapping between optical depth and geometric height in contrast to the extended and non-local formation of the chromospheric lines above it.

In order to compare the radiative-transfer formation heights with the inversion-derived geometric heights, we extract distributions of geometric height from the inversion pixel atmospheres at selected optical-depth ranges. Candidate intervals in  $\log \tau_{500}$  were scanned, and the interval for which the median inversion height was closest to the mode of the corresponding Lightweaver formation height probability density function was selected. The median reduces sensitivity to the extreme high altitude tail in the inversion heights, whilst the mode describes the central location of the Lightweaver distribution. This procedure therefore aligns representative central heights and does not fit the complete distributions. A fixed optical-depth window of width 0.5 in  $\log \tau_{500}$  was used throughout, comparable to the vertical extent over which the contribution functions are sensitive and allowing a sufficient number of pixels to form reliable statistics. In addition, candidate intervals were rejected where more than 20% of inversion-derived heights lay outside the vertical range sampled by the Lightweaver contribution functions, effectively requiring that at least 80% of the distribution is confined to radiatively sampled layers. The Lightweaver contribution-function PDFs place 99.5% of their integrated weight below  $\approx 2.2$  Mm and so this condition ensures that any  $\tau$  populations dominated by the spurious high-altitude contributions are excluded.

The matching procedure was applied separately to three representative spectral windows sampling the line core, inner wings, and outer wings. These correspond to CRISP tunings at line centre (0 pm), inner-wing offsets of  $\pm 28$  pm, and outer-wing offsets of  $\pm 59.5$  and  $\pm 73.5$  pm from the Ca ii 8542 line core. These wavelength positions were selected to demonstrate the chromospheric core response together with progressively deeper atmospheric layers in the inner and outer wings of the line.

The resulting comparisons between the Lightweaver formation-height probability density functions and the best-matching

inversion-derived height distributions are shown for each annulus in Figure 6, with the latter shown as adjoining histograms. The optical-depth ranges identified through this matching procedure are found to be consistent across all annuli. The Ca ii 8542 Å line core is best reproduced by inversion layers near  $\log \tau_{500} \approx -5$ , while the inner-wing positions correspond to layers around  $\log \tau_{500} \approx -2$  to  $-1$ , and the outer-wing positions sample depths close to  $\log \tau_{500} \approx 0$ . The selected optical depth intervals yield median inversion heights in good agreement with the modes of the Lightweaver formation-height distributions calculated for the representative atmospheres. The Lightweaver calculations do not return appreciable contributions above approximately 2,500 km. However, because the use of median atmospheres suppress pixel-by-pixel variation, these calculations do not sample the full range of responses that may occur within each annulus. The absence of extreme formation heights therefore suggests that such heights are not required by the representative stratifications, although it does not independently establish that every extreme inversion height is unphysical. Taken together, these findings support the interpretation that, within the modelled atmospheres considered here, the inversion height scale down to  $\log \tau_{500} \approx -5$  provides a consistent description of the atmospheric layers sampled by the line.

Since the radiative-transfer calculations are evaluated inside the inversion-derived atmospheres themselves, the comparison does not constitute an independent validation of the atmospheric stratification. Nevertheless, this comparison provides a direct model-dependent mapping between the inversion optical-depth layers and the effective geometric heights sampled by the Ca ii 8542 nm CRISP observations, allowing us to quantify the wavelength-dependent sensitivity of the line within the (inversion) atmospheres, which has not previously been demonstrated in a spatially resolved sunspot atmosphere. Establishing this consistency is particularly important for interpreting chromospheric wave signatures observed in Ca ii 8542 Å, which are known to vary systematically across the sunspot structure (Murabito et al., 2019).

These formation heights and optical depth ranges are broadly consistent with previous estimates of the Ca ii 8542 Å line-core formation height. For example, Cauzzi et al. (2008) place the line core primarily in the middle chromosphere at heights of order 1–1.5 Mm, while response-function calculations for quiet-Sun reference atmospheres in Noda et al. (2016) show the strongest sensitivity is concentrated around  $\log \tau_{500} \approx -5.5$  to  $-4$ , which of course maps to similar geometric heights depending on the atmospheric stratification. Likewise, using solar-filament models, Díaz Baso et al. (2019) report their peak chromospheric sensitivity near  $\log \tau_{500} \approx -5$  with a useful response extending over roughly  $\log \tau_{500} \approx -6$  to  $-4$ . These quoted values identify maxima in model-dependent response functions—i.e., sensitivity to perturbations—rather than optical depths at which the inversion is necessarily accurate or uniquely constrained. Within this context, the results presented here are therefore consistent with the Ca ii 8542 Å line sampling a broadly stable optical-depth range across the sunspot, while the associated geometric height varies systematically within the local inversion atmospheres through changes in the local  $\tau$ - $z$  mapping.

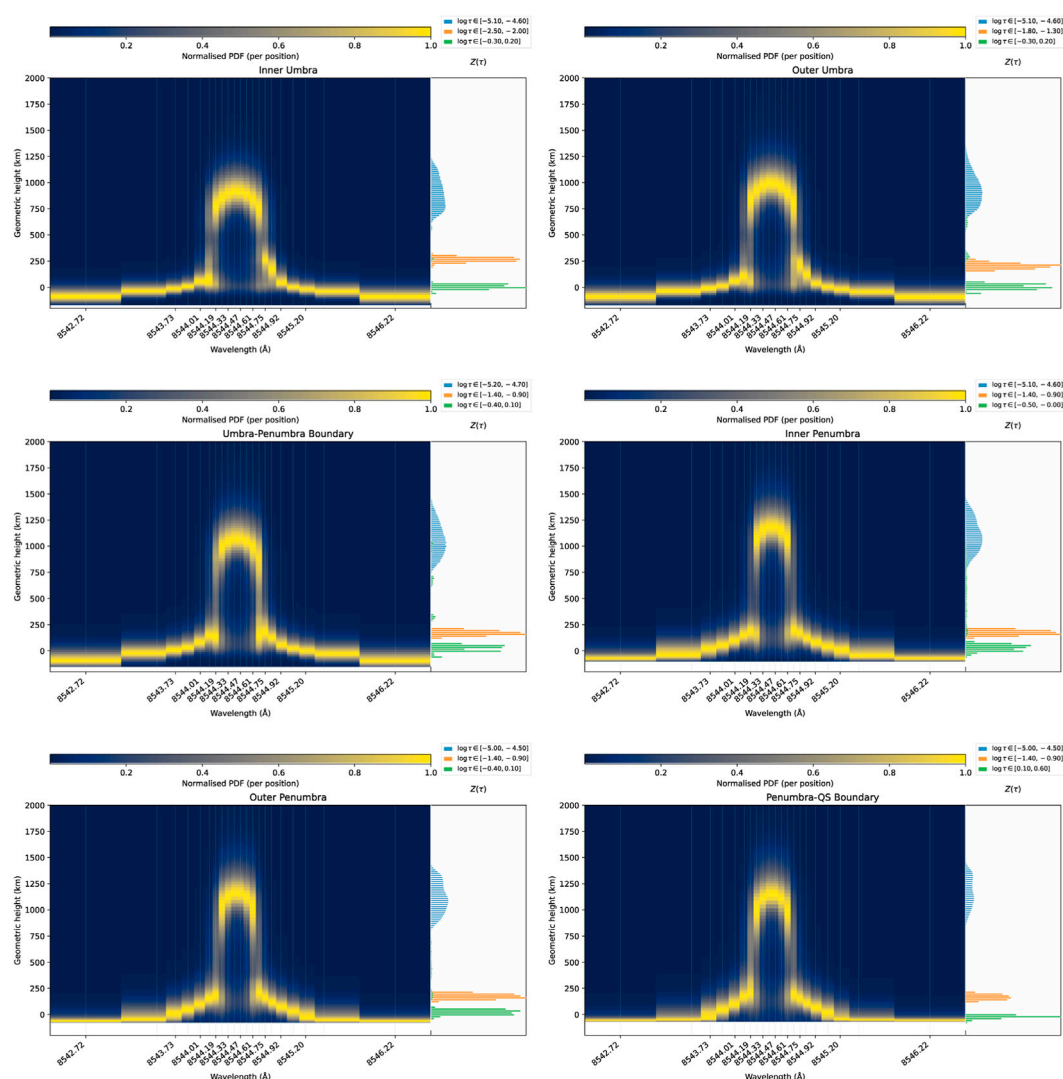


FIGURE 6

Comparison between Lightweaver formation height probability density functions and NICOLE geometric height distributions for each annulus. The displayed NICOLE distributions are restricted to heights from 0–2000 km for clarity. For each radial region, the main panel shows the CRISP-weighted formation-height probability density function as a function of wavelength, normalised independently at each wavelength position. The adjacent histograms depict the inversion-derived height distributions within the  $\log \tau_{500}$  bands that best match the modal formation heights. Each adjoining distribution is shown on a common horizontal scale from 0 to 1, normalised by the largest histogram value. The horizontal amplitude therefore represents the normalised distribution rather than raw pixel count, allowing the distribution shapes to be compared without dependence on the number of inversion pixels in each annulus. The wavelength positions correspond to CRISP tunings at line centre (0 pm), inner-wing offsets of  $\pm 28$  pm, and outer-wing offsets of  $\pm 59.5$  and  $\pm 73.5$  pm from the Ca ii 8542 Å line core.

## 5 Discussion and conclusion

We find the effective optical-depth range sampled by each CRISP spectral tuning by selecting the  $\log \tau_{500}$  interval whose geometric-height distribution best matches the modal height from the Lightweaver formation-height probability density function. Across all annuli the Ca ii 8542 Å line core is consistently associated with  $\log \tau_{500} \approx -5.0 \pm 0.3$ , corresponding to formation heights of approximately 0.9–1.2 Mm. The inner wings sample  $\log \tau_{500} \approx -1.5 \pm 0.5$  ( $\approx 150$ –300 km), while the far wings sample layers near  $\log \tau_{500} \approx 0$ .

The stability of the selected optical depth intervals, together with the radial variation in inversion derived geometric height, is consistent with differences in the local optical depth ( $\tau$ ) to geometric

height ( $z$ ) mapping between annuli (Murabito et al., 2019). Put another way, the radiative-transfer calculations shown here characterise the range of optical depths sampled by the line, while the inversion atmospheres determine how these layers translate into model geometric height. Together they provide a practical mapping of the inversion-derived height scale over the optical depth range sampled by Ca ii line.

The geometric heights should nevertheless be interpreted as model-dependent quantities within the NICOLE atmospheres used. The  $\tau$ – $z$  conversion inherits the assumption of hydrostatic equilibrium used in the modelling of the sunspot atmospheres by NICOLE (Socas-Navarro et al., 2015). Therefore potentially important contributions to the force balance by magnetic

pressure and tension forces are not accounted for in this work. A magnetohydrostatically (MHS) constrained inversion would provide a more complete determination of the  $\tau$ – $z$  mapping, for example, with the FIRTEZ-DZ code (Yabar et al., 2019), and would make an exciting extension to this work. Similarly, the simultaneous inversion of multiple spectral lines—as spearheaded by facilities such as DKIST—would provide stronger constraints on the atmospheric stratification and thereby reduce the model dependence of the inferred formation-height diagnostics. An independent test would require Ca ii 8542Å profiles synthesised from a radiation MHD sunspot simulation, followed by inversion of those profiles and direct comparison with the known simulation atmosphere.

Furthermore, another potentially significant effect which is neglected in this work is the Wilson depression, where the strong magnetic field of the sunspot depresses the  $\tau = 1$  surface. This likely introduces a spatially varying zero-point in geometric height between the modelled atmospheres of different annuli, which may partly contribute to the systematic variation with radius described above. However, such an effect would primarily shift the formation-height distributions in geometric height (for example, manifesting as a vertical offset in Figure 5) and is unlikely to substantially alter their overall shape. Whilst the mapping of contribution functions and optical-depth ranges onto absolute geometric height is subject to these uncertainties, our analysis remains useful for characterising the wavelength-dependent sensitivity of the line within each modelled atmosphere.

This study provides a model dependent mapping between Lightweaver contribution functions, inversion optical depth, and NICOLE geometric height across a sunspot. This mapping can support the interpretation of Ca ii 8542Å observations from SST and DKIST, particularly for studies of height dependent wave propagation, provided that the hydrostatic, atomic model, and median atmosphere limitations are retained. This connection will play an important role in the interpretation of high-resolution observations of the enigmatic chromosphere from facilities such as DKIST (Rimmele et al., 2020) and the SST (Scharmer, 2006), where the increasing capability of modern spectroscopic measurements demands a precise characterisation of the height ranges probed by their spectral lines, particularly for studies of height-dependent wave propagation in structured solar atmospheres.

## Data availability statement

The codes used for the analysis in this study can be found in the WaLSAtools repository. The data analysed in this study are subject to the WaLSA collaboration data policy and are therefore not publicly available. Access may be possible within the framework of the WaLSA collaboration and in coordination with colleagues at the University of Oslo. Requests to access the datasets should be directed to t.duckenfield@qub.ac.uk (<https://github.com/WaLSAteam/WaLSAtools>; Jafarzadeh et al., 2025).

## Author contributions

TJD: Conceptualization, Methodology, Software, Formal analysis, Investigation, Visualization, Writing – original draft. DBJ:

Conceptualization, Supervision, Funding acquisition, Writing – review and editing. SJ: Methodology, Validation, Writing – review and editing. LRvdV: Resources, Data curation, Writing – review and editing. VMJH: Methodology, Resources, Data curation, Writing – review and editing.

## Funding

The author(s) declared that financial support was received for this work and/or its publication. TJD and DBJ thank the support of the Leverhulme Trust via the Research Project Grant RPG-2019-371. TJD, DBJ and SJ wish to thank the United Kingdom Science and Technology Facilities Council (STFC) for the consolidated grants ST/T00021X/1 and ST/X000923/1. DBJ also acknowledges funding from the United Kingdom Space Agency via the National Space Technology Programme (grant SSc-009). SJ acknowledges support from the European Research Council under the European Union Horizon 2020 research and innovation program (grant agreement No. 682462) and from the Research Council of Norway through its Centres of Excellence scheme (project No. 262622).

## Acknowledgments

We wish to acknowledge scientific discussions with the Waves in the Lower Solar Atmosphere (WaLSA; [www.WaLSA.team](http://www.WaLSA.team)) team, which has been supported by the Research Council of Norway (project no. 262622), The Royal Society (award no. Hooke18b/SCTM; Jess et al., 2021), and the International Space Science Institute (ISSI Team 502).

## Conflict of interest

The author(s) declared that this work was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

## Generative AI statement

The author(s) declared that generative AI was not used in the creation of this manuscript.

Any alternative text (alt text) provided alongside figures in this article has been generated by Frontiers with the support of artificial intelligence and reasonable efforts have been made to ensure accuracy, including review by the authors wherever possible. If you identify any issues, please contact us.

## Publisher's note

All claims expressed in this article are solely those of the authors and do not necessarily represent those of their affiliated

organizations, or those of the publisher, the editors and the reviewers. Any product that may be evaluated in this article, or claim that may be made by its manufacturer, is not guaranteed or endorsed by the publisher.

## References

- Carlsson, M., and Stein, R. F. (1997). Formation of solar calcium h and k bright grains. *Astrophysical J.* 481, 500–514. doi: 10.1086/304043
- Cauzzi, G., Reardon, K. P., Uitenbroek, H., Cavallini, F., Falchi, A., Falciani, R., et al. (2008). The solar chromosphere at high resolution with ibis. *Astronomy and Astrophysics* 480, 515–526. doi: 10.1051/0004-6361:20078642
- de la Cruz Rodríguez, J., Löfdahl, M. G., Sütterlin, P., Hillberg, T., and Rouppe van der Voort, L. (2015). CRISPRED: a data pipeline for the CRISP imaging spectropolarimeter. *A&A* 573, A40. doi: 10.1051/0004-6361/201424319
- Díaz Baso, C. J., Martínez González, M. J., Asensio Ramos, A., and de la Cruz Rodríguez, J. (2019). Diagnostic potential of the ca8542 Å line for solar filaments. *A&A* 623, A178. doi: 10.1051/0004-6361/201834793
- Jafarzadeh, S., Jess, D. B., Stangalini, M., Grant, S. D. T., Higham, J. E., Pessah, M. E., et al. (2025). Wave analysis tools. *Nat. Rev. Methods Primers* 5, 21. doi: 10.1038/s43586-025-00392-0
- Jess, D. B., Morton, R. J., Verth, G., Fedun, V., Grant, S. D. T., and Giagkiozis, I. (2015). Multiwavelength studies of MHD waves in the solar chromosphere. An overview of recent results. *Space Sci. Rev.* 190, 103–161. doi: 10.1007/s11214-015-0141-3
- Jess, D. B., Keys, P. H., Stangalini, M., and Jafarzadeh, S. (2021). High-resolution wave dynamics in the lower solar atmosphere. *Philosophical Trans. R. Soc. Lond. Ser. A* 379, 20200169. doi: 10.1098/rsta.2020.0169
- Jess, D. B., Jafarzadeh, S., Keys, P. H., Stangalini, M., Verth, G., and Grant, S. D. T. (2023). Waves in the lower solar atmosphere: the dawn of next-generation solar telescopes. *Living Rev. Sol. Phys.* 20, 1. doi: 10.1007/s41116-022-00035-6
- Leenaarts, J., Carlsson, M., and van der Voort, L. R. (2012). The formation of the hα line in the solar chromosphere. *Astrophysical J.* 749, 136. doi: 10.1088/0004-637X/749/2/136
- Leenaarts, J., Pereira, T. M. D., Carlsson, M., Uitenbroek, H., and Pontieu, B. D. (2013). The formation of iris diagnostics. ii. the formation of the mg ii h&k lines in the solar atmosphere. *Astrophysical J.* 772, 90. doi: 10.1088/0004-637X/772/2/90
- Murabito, M., Ermolli, I., Giorgi, F., Stangalini, M., Guglielmino, S. L., Jafarzadeh, S., et al. (2019). Height dependence of the penumbral fine-scale structure in the inner solar atmosphere. *Astrophysical J.* 873, 126. doi: 10.3847/1538-4357/aaf727
- Noda, C. Q., Shimizu, T., de la Cruz Rodríguez, J., Katsukawa, Y., Ichimoto, K., Anan, T., et al. (2016). Spectropolarimetric capabilities of caii 8542 Å line. *Mon. Notices R. Astronomical Soc.* 459, 3363–3376. doi: 10.1093/mnras/stw867
- Osborne, C. M. J., and Milić, I. (2021). The lightweaver framework for nonlocal thermal equilibrium radiative transfer in python. *Astrophysical J.* 917, 14. doi: 10.3847/1538-4357/ac02be
- Rimmele, T. R., Warner, M., Keil, S. L., Goode, P. R., Knölker, M., Kuhn, J. R., et al. (2020). The daniel k. inouye solar telescope – observatory overview. *Sol. Phys.* 295, 172. doi: 10.1007/s11207-020-01736-7
- Rodríguez-Gómez, J. M., Kuckein, C., Manrique, S. J. G., Saqri, J., Veronig, A., Gömöry, P., et al. (2024). The plasma in quiet sun regions: Multi-instrument view. *Astrophysical J.* 964, 27. doi: 10.3847/1538-4357/ad1f64
- Rouppe van der Voort, L. H. M., De Pontieu, B., Carlsson, M., de la Cruz Rodríguez, J., Bose, S., Chintzoglou, G., et al. (2020). High-resolution observations of the solar photosphere, chromosphere, and transition region. A database of coordinated IRIS and SST observations. 641, A146. doi: 10.1051/0004-6361/202038732
- Scharmer, G. B. (2006). Comments on the optimization of high resolution fabry-pérot filtergraphs. *Astronomy and Astrophysics* 447, 1111–1120. doi: 10.1051/0004-6361:20052981
- Scharmer, G. B., Bjelksjö, K., Korhonen, T. K., Lindberg, B., and Petterson, B. (2003). “The 1-meter Swedish solar telescope,” in *Innovative Telescopes and Instrumentation for Solar Astrophysics*. Editors S. L. Keil, and S. V. Avakyan, 341–350. doi: 10.1117/12.460377
- Scharmer, G. B., Narayan, G., Hillberg, T., de la Cruz Rodríguez, J., Löfdahl, M. G., Kiselman, D., et al. (2008). Crisp spectropolarimetric imaging of penumbral fine structure. *Astrophysical J.* 689, L69–L72. doi: 10.1086/595744
- Socas-Navarro, H., De La Cruz Rodríguez, J., Ramos, A. A., Bueno, J. T., and Cobo, B. R. (2015). An open-source, massively parallel code for non-lte synthesis and inversion of spectral lines and zeeman-induced stokes profiles. *Astronomy and Astrophysics* 577, A7. doi: 10.1051/0004-6361/201424860
- Uitenbroek, H. (2001). Multilevel radiative transfer with partial frequency redistribution. *Astrophysical J.* 557, 389–398. doi: 10.1086/321659
- Van Noort, M., Rouppe Van Der Voort, L., and Löfdahl, M. G. (2005). Solar image restoration by use of multi-frame blind de-convolution with multiple objects and phase diversity. *Sol. Phys.* 228, 191–215. doi: 10.1007/s11207-005-5782-z
- Wu, Y., Li, W., Bai, X., Chen, F., Li, H., and Deng, Y. (2025). Characterization and formation of the mg i 12.32 m line in the quiet sun and sunspot. *Astronomy and Astrophysics* 703, A48. doi: 10.1051/0004-6361/202556092
- Yabar, A. P., Borrero, J. M., and Cobo, B. R. (2019). Firtez-dz. a forward and inverse solver of the polarized radiative transfer equation under zeeman regime in geometrical scale. *Astronomy and Astrophysics* 629, A24. doi: 10.1051/0004-6361/201935692

## Supplementary material

The Supplementary Material for this article can be found online at: <https://www.frontiersin.org/articles/10.3389/fspas.2026.1873920/full#supplementary-material>